Some Conceptual Misunderstandings of the Fundamentals of Soft Set Theory

1Prof. D. Singh (Former Professor, Indian Institute of Technology, Bombay), 2Onyezili I. A. (Corresponding author)
1Mathematics Department, Ahmadu Bello University, Zaria, Nigeria, 2Department of Mathematics, University of Abuja, Nigeria
Email : {1mathdss@yahoo.com, 2ijeozili@gmail.com}

ABSTRACT
Our main objective in this paper is to clarify some conceptual misunderstandings of the fundamentals of soft set theory in particular reference to the recent papers of Fu Li (Notes on the soft operations, ARPN Journal of systems and software, 1(2011) 205 – 208) and Xun Ge and S. Yang (Investigations on some operations of soft sets, World Academy of Sciences, Engineering and Technology, 75 (2011) 1113 – 1116.

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1. INTRODUCTION
In 1999, Molodtsov [1] introduced soft sets with an objective to solve a class of problems involving uncertainties for which other extant modern theories viz; probability theory, fuzzy set theory, theory of vague sets theory of interval mathematics, theory of rough sets, etc., are found incapacitated, largely due to their inherent parameterization inadequacies.

Owing to its rich potential for multi-directional applications, intensive studies, particularly towards systematization of the fundamentals of soft set theory (SST, for short) have been undertaken during the recent years.

Subsequent to Molodtsov [1], Maji et al. [2] were the first to define a number of operations on soft sets and established many of their properties.

Fu Li [3] put forward a counter example to proposition 2.1 (1 – 3) of [2] and established some distributive laws.


As mentioned in the abstract, this paper intends to show that the explications of Fu Li [3] and Ge and Yang [5] are not true.

2. MAIN FINDINGS
Following [1] and other subsequent works, a soft set can be defined as follows:
Let of U be an initial universe set and E be the set of all possible parameters under consideration with respect to U.

Let P(U) be the power set of U and A ⊆ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

Characteristically, F is a set-valued function of a set. The soft set (F, A) over U is a parameterized family \( \{ F(\varepsilon_i) : \varepsilon_i \in A, i = 1, 2, ..., k \} \) of subsets of the universe set U. For each \( e \in A \), F(e) may be considered as the set of \( e \approx \) approximate elements of the soft set.

It is important to note that a soft set is not a set and consequently, some operations defined in SST need not have their counterparts in set theory (ST, for short). For example, the Not set operator, denoted \( \overline{\cdot} \), defined in...
the context of soft sets in [2], does not have its counterpart in ST.

2.1 Some Conceptual Misunderstandings appearing in [3] and [5]

The counter example to proposition 2.1 (1–3) of [2], provided in [3], is misconstrued. The latter has not taken into account that the two unary operations \( \neg \) (defined on a set of parameters) and \( \neg \) (defined on an individual parameter) are different ([2], P.557) and goes on to proving its false claim using De Morgan’s laws for sets, along with numerous apparent errors appearing throughout. Moreover, proposition 2.5 of [3] has been already proved incorrect in [4], and our own analytic proof in 2.2 below.

Let us consider the counter example used in ([3] P. 205):

Let \( X = \{a, b, c, d\}, A = \{a, b, c\}, B = \{b, d\}, A \cup B = \{a, b, c, d\} \). Then \( \neg A \) (as defined in [2]) is \( \{\neg a, \neg b, \neg c\} \) and not the set \( \{d\} \) (as in [3]). Accordingly, for example, \( \neg A \cup \neg B = \{\neg a, \neg b, \neg c, \neg d\} \neq \neg (A \cup B) \) is correct. Similar observations apply to all other results of proposition 2.1 [2] to conclude their correctness. In fact, the counter example considered in [3] is not in place at all. At its best, given \( X = \{a, b, c, d\} \), the set of all parameters, and \( A = \{a, b, c\} \), the set \( \{d\} = A^c \) in \( X \), the relative complement of \( A \), defined in ST in which \( \neg A \) is not a well formed formula.

We need to emphasize that \( \neg A \) (the Not set of the set \( A \) of parameters) is different from the complement of the soft set \((F, A)\), denoted \((F, A)^c = (F^c, \neg A)\), where \( F^c: \neg A \rightarrow P(U) \) is a mapping given by \( F^c(\alpha) = U - F(\neg \alpha) \), for all \( \alpha \in \neg A \).

For illustration, let us consider the example 2.1 of [2]:

Let \( U = \{h_1, h_2, h_3, h_4, h_5, h_6\} \) be the set of all houses for consideration, and \( E = \{e_1, e_2, e_3, e_4, e_5\} \) be the set of all possible parameters with respect to \( U \).

Let \( A = \{e_1\} \), where \( e_1 \) denotes the predicate name expensive houses which are \( h_2 \) and \( h_4 \). Then, \( F(e_1) = \{\text{houses (expensive)}\} = \{h_2, h_4\} \), the functional – value corresponding to the predicate name expensive houses, and the soft set \((F, A) = \{\text{expensive houses} = \{h_2, h_4\}\} \).

Also, \( \neg A = \{\neg e_1\} \subseteq \neg E \), and

\[
F^c(\neg e_1) = U - F(\neg (\neg e_1)) = U - F(e_1)
\]

\[
= U - \{h_2, h_4\}
\]

\[
= \{h_1, h_3, h_5, h_6\} \subset U.
\]

Clearly, \( F(\neg e_1) \) is not defined, rather it is \( F^c(\neg e_1) \) which is defined.

Note that \( \neg \) - operator is defined on a set of parameters but not on soft sets. In fact, it is the complementation which is defined on soft sets.

Further, in view of the intended interpretation of Not set discussed above, it follows that the Assumption: Not – set of each set of parameters is a subset of \( E \) of [5] is untenable.

In the example taken above, where \( \neg A = \{\neg e_1\}, \neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4, \neg e_5\} \),

\( \neg A \not\subset \neg E \) and \( \neg A \not\subset E \).

2.2 Proof of incorrectness of 2.5 of [3]

Lemma

Let \( A, B \) and \( C \) be sets, then

(i) \( \{A - (B \cup C)\} \cap \{(A \cup B) - (A \cup C)\} = \phi; \)

(ii) \( \{(B \cup C) - A\} \supset \{(A \cup C) - (A \cup B)\}; \)

(iii) \( \{A \cap (B \cup C)\} \subset \{A \cup (B \cap C)\}. \)

Proof: Easy.

Proposition 2.5 [3]:

Let \((F, A), (G, B) \) and \((H, C)\) be soft sets over a common universe \( U \), then

\[
(i) \quad (F, A) \cup ((G, B) \cap_e (H, C)) = ((F, A) \cup (G, B)) \cap_e ((F, A) \cup (H, C));
\]

\[
(ii) \quad (F, A) \cap_e ((G, B) \cup (H, C)) = ((F, A) \cap_e (G, B)) \cup ((F, A) \cap_e (H, C)).
\]
Proof:

Let \((G, B) \cap_e (H, C) = (L, D)\), where \(D = B \cup C\) and \(\forall d \in D\),

\[
L(d) = \begin{cases} 
  G(d), & \text{if } d \in B - C; \\
  H(d), & \text{if } d \in C - B; \\
  G(d) \cap H(d), & \text{if } d \in B \cap C.
\end{cases}
\]

Let

\[
(F, A) \sqcup (L, D) = (M, K),
\]

where \(K = A \cup D = A \cup (B \cup C) = (M, A \cup (B \cup C)), \forall k \in K.\)

Using definitions of \(\cap_e\) and \(\sqcup\) in [4], we have:

\[
M(k) = \begin{cases} 
  F(k), & \text{if } k \in A - (B \cup C) \\
  = (A - B) \cap (A - C); \\
  L(k), & \text{if } k \in (B \cup C) - A \\
  = (B - A) \cup (C - A); \\
  F(k) \cup L(k), & \text{if } k \in A \cap (B \cup C) \\
  = (A \cap B) \cup (A \cap C). 
\end{cases}
\]

Let \((F, A) \sqcup (G, B) = (H_1, A \cup B)\)

and

\((F, A) \sqcup (H, C) = (H_2, A \cup C)\).

Then, \(((F, A) \sqcup (G, B)) \cap_e ((F, A) \sqcup (H, C)) = (H_1, A \cup B) \cap_e (H_2, A \cup C)\).

Let \((H_1, A \cup B) \cap_e (H_2, A \cup C) = (H_3, (A \cup B) \cup (A \cup C)) = (H_3, A \cup B \cup C)\).

Then, we have \(\forall k \in A \cup B,\)

Hence, by Lemma, \(M(k) \neq H_3(k), \forall k \in A \cup B \cup C,\) and the assertion 2.5 (i) of [3] does not hold, in general.

Similarly, it can be proved that the assertion 2.5 (ii) [3] does not hold too.

3. CONCLUDING REMARKS

As is known, a number of works have recently appeared that critically examine Maji et al [2], point out its other inconsistencies and provide some new operations on soft sets and their properties. We believe that the content of this paper would help in clarifying some further related issues. We hope to address many other fundamental issues currently being discussed in our forthcoming paper.
REFERENCES


