Notes on the soft operations

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ABSTRACT

Soft set theory can be seen as a new mathematical approach to vagueness, authors P.K.Maji et al. introduced some operations and gave their some properties on soft sets, and in inference [12], authors defined some new operations and discussed their properties on soft sets. In this paper, we first point a small error in inference [4] and correct it; furthermore, based on some results of soft operations, using the DeMorgan's laws, we give the distributive laws of the restricted union and the restricted intersection and the distributive laws of the union and the extended intersection.

Keywords: soft set, soft operation, extended intersection, distributive law.

1. INTRODUCTION

In 1999, Molodsov [1] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodsov in [1]. After Molodsov's work, some different applications of soft sets were studied in [2,3]. Furthermore, Maji, Biswas and Roy worked on soft set theory in [4]. Also Maji et al. [5] presented the definition of fuzzy soft set and Roy et al. presented some applications of this notion to decision making problems in [6].

Recently, the many authors discuss the soft set, research on the soft set theory is progressing rapidly, for example, the concepts of soft semi-ring, soft group, soft BCK/BCI-algebra, soft BL-algebra, and fuzzy soft group etc. have been proposed and investigated (see [7-11] respectively). M.Irfan Ali et.in [12] discussed new operations in soft set theory which the authors gave the definition of the restricted intersection, the restricted difference and extended intersection of soft sets, and gave the DeMorgan's law in soft set theory.

In this paper, we first point the some small errors in inference [4] and correct it, and using the DeMorgan's law, we give their distributive laws. The rest of the paper is organized as following, in section 2, we give some definitions and some results of soft sets which we will use in this paper, and point an small error in the inference [4]. In section 3, we discuss the distributive laws of soft operations, conclusions are given in section 4.

2. PREMERILARY

Definition 1.1[4] Let \( U \) be an initial universe set and \( E \) be a set of parameters. Let \( P(U) \) denotes the power set of \( U \) and \( A \subseteq E \). Then a pair \((F,A)\) is called a soft set over \( U \), where \( F \subseteq P(U) \) is a mapping.

That is, the soft set is a parameterized family of subsets of the set \( U \). Every set \( F(e), \forall e \in E \), from this family may be considered as the set of \( e \)-elements of the soft set \((F,E)\), or considered as the set of \( S \)-approximate elements of the soft set. According to this manner, we can view a soft set \((F,E)\) as consisting of collection of approximations: \( (F,E) = \{F(e) | e \in E\} \).

Definition 1.2[4] Let \( E = \{e_1, e_2, \ldots\} \) be a set of parameters. The NOT set of \( E \) denoted \( \neg E \) is defined by \( \neg E = \{-\neg e_1, -\neg e_2, \ldots\} \).

Definition 1.3[4] The complement of soft set \((F,A)\) is denoted by \((F,A)^c\) and is defined by \((F,A)^c = (F^c, \neg A)\), where \( F^c : \neg A \rightarrow P(U) \) is a mapping given by \( F^c(\alpha) = U - F(\neg \alpha), \neg \alpha \in A \).

Proposition 1.4[4] Let \( E \) be a set of parameters and \( A, B \subseteq E \). Then

(i) \((-(-A)) = A\); (ii) \((-A \cup B) = -A \cup -B\); (iii) \((-A \cap B) = -A \cap -B\).

The results (ii) and (iii) are not right. For example:
Let \( X = \{a,b,c,d\}, A = \{a,b,c\}, B = \{b,d\}\). By definition 1.2, \(-A = \{d\}\), \(-B = \{a,c\}\), \(A \cup B = \{a,b,c,d\}\), \(A \cap B = \{b\}\), but, \(-A \cup -B = \emptyset \neq -A \cup -B\).
We can revise it as:

**Proposition 1.5** Let $E$ be a set of parameters and $A, B \subseteq E$. Then

\[
- (A \cup B) = -A \cap -B; \quad (ii) \quad - (A \cap B) = -A \cup -B
\]

**Proof:** \(\forall x \cup -(A \cup B)\) \iff \(x \notin A \cup B\) \iff \(x \notin A\) and \(x \in B\) \iff \(x \notin -A \) and \(x \in -B\) \iff \(x \in (\neg A \cap \neg B)\). The other is similar.

**Definition 1.6** The union of two soft sets \((F, A)\) and \((G, B)\) over a common universe \(U\) is the soft set \((H, C)\), where \(C = A \cup B\), \(\forall e \in C\), denoted as \((F, A) \wedge (G, B) = (H, C) = (A \cup B, B),\) where

\[
H(e) = \begin{cases} 
F(e), & \text{if } e \in A - B \\
G(e), & \text{if } e \in B - A \\
F(e) \cup G(e), & \text{if } e \in A \cap B 
\end{cases}
\]

**Definition 1.7** The restricted intersection of two soft sets \((F, A)\) and \((G, B)\) over a common universe \(U\) is the soft set denoted as \((F, A) \cap (G, B)\) and is defined as \(F, A \cap (G, B) = (H, C)\), where \(C = A \cap B\), and \(\forall e \in C\), \(H(c) = F(c) \cap G(c)\).

**Definition 1.8** The extended intersection of two soft sets \((F, A)\) and \((G, B)\) over a common universe \(U\) is the soft set \(H, C\), where \(C = A \cup B\), and \(\forall e \in C\), denoted as \((F, A) \wedge \land (G, B) = (H, C) = (A \cup B, B),\) where

\[
H(e) = \begin{cases} 
F(e), & \text{if } e \in A - B \\
G(e), & \text{if } e \in B - A \\
F(e) \cap G(e), & \text{if } e \in A \cap B 
\end{cases}
\]

**Definition 1.9** Let \((F, A)\) and \((G, B)\) be two soft sets over a common universe \(U\) such that \(A \cup B \neq \emptyset\). The restricted union of \((F, A)\) and \((G, B)\) is denoted as \((F, A) \cup (G, B)\), and is defined as \((F, A) \cup (G, B) = (H, C)\) and, where \(C = A \cup B\), and \(\forall e \in C\), \(H(c) = F(c) \cup G(c)\).

3. THE NOTE OF THE SOFT

**OPERATIONS**

**Theorem 2.1** Let \((F, A), (G, B)\) be two soft sets over the common universe \(U\), such that \(A \cap B \neq \emptyset\), then

\[
(i) \quad ((F, A) \cup (G, B)) = (F, A) \cap (G, B)
\]

**Proposition 2.2** Let \((F, A), (G, B), (H, C)\) be soft sets in the common universe \(U\), then

\[
(i) \quad (F, A) \cup (G, B) \subseteq (H, C) = (L, D) = (L, B \cap C),
\]

\(\forall d \in D\)

\[
L(d) = B \cap C(d) = G(d) \cap H(d)
\]

and called the distributive law of \(\cup_R\)

and \(\cap_R\) with respect to \(\cap\).

**Proof:**

Let \((G, B) \subseteq_R (H, C) = (L, D) = (L, B \cap C),\)

\(\forall d \in D\)

\[
L(d) = B \cap C(d) = G(d) \cap H(d)
\]

and let \((F, A) \subseteq_R (L, D) = (M, K) = (M, A \cap D),\)

\(\forall k \in K = A \cap D,\)

\[
M(k) = F(k) \cup L(k) = F(k) \cup (G(k) \cap H(k)) = F(k) \cup G(k) \cap (H(k))
\]

then \(\forall k \in M(k)\) if and only if \(\forall k \in (F(k) \cup G(k)) \cap (F(k) \cup H(k))\) if and only if \(\forall k \in F(k) \cup G(k)\), and \(\forall k \in F(k) \cup H(k)\) if and only if \(\forall k \in (F, A) \cup_R (G, B)\) and \(\forall k \in (F, A) \cup_R (H, C)\) if and only if \(\forall k \in ((F, A) \cup_R (G, B)) \cap ((F, A) \cup_R (H, C))\).

Similarly, we can proof the other equation.

**Proposition 2.3** Let \(A, B, C\) be sets, then

\[
(i) \quad (A \cup B) - C = (A - C) \cup (B - C)
\]

\(\forall d \in D\)

\[
A - (B \cup C) = (A - B) \cup (A - C)
\]
Proof \[(i) \quad \forall x \in (A \cup B) - C \quad \text{if and only if} \quad x \in A \cup B, \text{and} \quad x \not\in C \quad \text{if and only if} \quad x \in A \text{ or } x \in B \text{ and } x \not\in C \quad \text{if and only if} \quad (x \in A \text{ and } x \not\in C) \text{ or } (x \in B \text{ and } x \not\in C) \text{ if and only if } x \in A - B \text{ or } x \in B - C \text{ if and only if } x \in (A - C) \cup (B - C).
\]

\[(ii) \quad \forall x \in (A \cap B) - C \quad \text{if and only if} \quad x \in A \cap B, \text{and} \quad x \not\in C \quad \text{if and only if} \quad x \in A \text{ and } x \not\in B \text{ and } x \not\in C \quad \text{if and only if} \quad (x \in A \text{ and } x \not\in B) \text{ and } (x \in A \text{ and } x \not\in C) \text{ if and only if } x \in (A - B) \cap (A - C) \text{ if and only if } x \in (A - C) \cap (B - C).
\]

\[\begin{align*}
F(k), & \quad \text{if } k \in A - (B \cup C) = (A - B) \cap (A - C) \\
L(k), & \quad \text{if } k \in (B \cup C) - A = (B - A) \cup (C - A) \\
G(k) \cap H(k), & \quad \text{if } k \in A \cap (B \cup C) = (A \cap B) \cup (A \cap C)
\end{align*}\]

Let
\[\begin{align*}
H_1(k) = \begin{cases} F(k), & \text{if } k \in A - B \\ G(k) \cap G(k), & \text{if } k \in A \cap B \end{cases}
\]

\[\begin{align*}
H_2(k) = \begin{cases} G(k), & \text{if } k \in B - C \\ G(k) \cap H(k), & \text{if } k \in B \cap C \end{cases}
\]

And \(\forall k \in B \cup C\), having

\[\begin{align*}
H_3(k) = \begin{cases} H_1(k), & \text{if } k \in (A - B) \cup (A - C) \\ H_2(k), & \text{if } k \in (A - C) \cup (A - B) \\ H_1(k) \cap H_2(k), & \text{if } k \in A \cup (B \cap C) \end{cases}
\]

Next, we need discuss
\[\begin{align*}
\forall k \in A - (B \cup C), k \in (B \cup C) - A & \quad \text{and} \quad k \in A \cap (B \cup C) \text{ respectively, by the definition of } \sim \text{ and } \hat{\hat{\tau}} \text{ again, in any case, we all can prove} \\
(F, A) \sim (G, B) \hat{\hat{\tau}} (H, C) & \equiv ((F, A) \sim (G, B)) \hat{\hat{\tau}} ((F, A) \sim (H, C))\text{ holds. That is, we prove that the equation (i) holds. Similarly, we can prove the other equation holds, too.}
\]

4. CONCLUSION

In this paper, we first point the some small errors in inference [4] and correct it, gave the right relation of \(\sim \cup \cap \cap\) on soft sets, and, by means of the DeMorgan's laws which introduced in [12], we discuss...
the distributive laws of $\bigcup_\mathcal{R}$ and $\mathfrak{R}$ with respect to $\mathfrak{F}$ and
the distributive laws of $\mathfrak{F}$ and $\mathfrak{G}$.

REFERENCES


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